Hybrid diversity strategy using MIMO radar for target tracking

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***Abstract*—For the development of radar systems with enhanced performance in target detecting and tracking abilities, various research groups have been investigating multiple input multiple output (MIMO) based radar systems. They have shown that MIMO radars perform better compared to conventional systems like bistatic or even SIMO radars. Especially MIMO capitalizes several diversity parameters inherent to its design to give higher resolution in both range and range rate. Several papers address these diversities. The current work aims at parameterizing these diversities especially spatial, waveform and frequency diversities and hence designing a system which employees a the resultant diversity constraints on the transmitter architecture and on the type and properties of transmit signals. As done in conventional system analysis, here also the formulation of ambiguity function is addressed and in the process a new tool called *ambiguity function pair* is developed. It has been shown with example evaluations that this ambiguity function pair is more mature in analyzing and tracking targets with systems employing diversity constraints. In the process of evaluation a new strategy with adaptive framework for tracking targets and getting higher range resolution and is addressed.**

I. Introduction

Multiple input multiple output (MIMO) radar systems have gained popularity and attracted attention of late for their ability to enhance system performance. In many ways MIMO radar is similar to MIMO communication system. Among the many possible uses of a radar system, tracking and detecting targets, estimating target model parameters and creating images of targets are some of the most common. Various authors have shown how MIMO radar enhances the performance [1-3].

As mentioned above of the many purposes of radar systems, tracking and detecting targets is one of the most common and important. In this regard the concept of diversity is discussed and several methods to improve the performance are reported. From all the analysis carried out so far, four types of diversities can be identified: spatial diversity - imposing architectural constraints on transmitter and receiver [3-4], waveform diversity - identifying several types of signals for transmission [5] and frequency diversity - spectrally separating information to be gathered [6]. Some of these have been widely discussed and system structures have been designed meeting this improved performance improvement. Lehmann et. al. have shown that coherent processing of signals has better resolution than for non coherent processing [7]. Antonio et. al. show that orthogonal input signals track the target better than coherent input signals [8]. Spatial diversity parameter has been discussed by Lehmann et. al [3]. Angular diversity strategies are discussed by Fishler et. al. [4]. This paper combines these ideas each of which corresponds to different diversities and develop new tools and strategies to adaptively track the target position and velocity. Classical ambiguity function introduced by Woodward is used to characterize the local and global resolution properties of time-delay and Doppler for narrow band waveforms. It has been extended for characterization of MIMO radar [10, 11]. We try to formulate the same for our model and in the process a new tool, *ambiguity function pair* is developed which has enhanced performance in tracking targets adaptively and giving higher range resolution.

The remainder of this paper is organized as follows. Section II introduces the MIMO radar system and models the signal and ambiguity function. Section III parameterizes the diversity constraints and introduces a new system architecture with its analysis. Section IV discusses the results and Section V concludes the paper.

II. MIMO Radar System

This section describes the formulation of MIMO transmitter, receiver and target architectures and transmit signal is modeled. Then it briefly describes the parametrization of ambiguity function for MIMO radar system. Thorough analysis of the properties of this function has already been addressed [9], and based on them various strategies of formulating the function are developed [10, 11] but here we adopt a design which correlates itself our system model discussed next [8].

A. System Model

MIMO radar as its name suggests comprises of multiple transmitters, the transmitter block and multiple receivers, the receiver block. Here a uniform linear array geometry (ULA) is considered these blocks. Target is assumed to have multiple scatterers aligned in a similar ULA format. The complete system has M transmitters and N receivers placed equidistantly at d_t and d_r respectively. Target has P scatterers where $\Delta_{i,j}$ is the distance between *i*th and *j*th scatterers.

Trying to parameterize the system we focus on separating the target effect from the effects of antenna structure and of the propagation between transmitters and target and between target and receivers [8]. Considering narrow band waveforms for transmission, reflectivity of target scatterers is modeled by zero-mean, unit-variance per dimension, independent and identically distributed Gaussian complex random variables. Hence the target can be modeled by a diagonal matrix of these variables, $\Sigma = [\zeta_{i,j}]$ with diagonal elements as random variables. Signals radiated from M transmitters illuminate the

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P target scatterers at $\phi_{m,p}$ where m = 0,1,...$M-1$ and q = 0,1,...$P-1$ and arrive at the receiver at $\theta_{n,p}, n$ = 0,1,....N-1. Assuming that the target array and receiver array are small compared to **R**, the average distance of target from transmitter block, $\phi_{m,p} = \phi_m$ and $\theta_{n,p} = \theta \forall m, n, p$. Hence the signal vector induced my *m*th transmit antenna is

$$g_m = [1, e^{-j2\Pi \sin \phi_m \Delta/\lambda}, ..., e^{-j2\Pi \sin \phi_m (P-1)\Delta/\lambda}]^T \quad (1)$$

where all Δ is the spacing between adjacent scatterers, λ is the carrier wavelength and the super script T denotes the vctor/matrix transposition. The phase shifts of the signal intercepted can be summed up as

$$k(\theta) = [1, e^{j2\Pi \sin \theta \Delta/\lambda}, ..., e^{j2\Pi \sin \theta (P-1)\Delta/\lambda}]^T \quad (2)$$

and the arriving signal excites the elements of the receiver array with phase shifts given by

$$a(\theta = [1, e^{-j2\Pi \sin \theta d_r/\lambda}, ..., e^{-j2\Pi \sin \theta (N-1) d_r/\lambda}]^T \quad (3)$$

where d_r is the inter element spacing in the receiver block.

The target model matrix and the vectors g_m, $k(\theta)$ and $a(\theta)$ completely describe the system and these parameters will remain independent of the input transmit signals [3].

B. Signal Model and Ambiguity Function

For a system architecture configured as above signal model design and ambiguity function formulation have been discussed by Lehmann et. al. [8], here is a brief outlay, based on which our further analysis develops. The transmit signal matrix is $s = [s_0 s_1 .... s_M]^T$ and the received signal is given as

$$r = a(\theta)k^T(\theta)\Sigma \sum_{m=0}^{M-1} g_m s_m + V = K\Sigma G s + V \quad (4)$$

where the matrix $K = a(\theta)k^T(\theta)$ and G represent the propagation delays along transmitter to target and target to receiver paths respectively. V is the complex Gaussian random variable matrix representing the additive noise along the signal path. Assuming that transmit signals are of length L samples. The angle θ can be estimated at the receiver by the term $\sum_{l=1}^{L} |a^H(\theta')r_l|^2$.

For the case of M transmitters and N receivers we totally have MN signals received. These MN signals are embedded in the matrix $r = [r_0 r_1 .... r_N]^T$, where each row represents the signal received by *n*th receiver and each r_n represents the reflection from target of all the M transmitted signals. The matrices and vectors K, Σ, G parameterize propagation delays, channel and target properties from transmitter to receiver blocks, in the sense $r(t)$ completely describes the MIMO system with s as the transmit signal matrix. Hence we can directly write the ambiguity function as

$$\Psi(\tau, f_r) = \int_{-\infty}^{\infty} r(t) r^*(t-\tau) e^{j2\Pi f_r t} dt \quad (5)$$

where τ and f_r represent the delay and doppler shift which can be expressed in terms of range and range rate.

III. Ambiguity Function Pair Model

In this section we introduce a new methodology for target tracking by combining the spatial, waveform and frequency diversity parameters. Spatial diversity attributes to the system model, in the sense, it puts constraints on the transmitter and receiver architecture and transmit signal type is not effected by this. Where as the waveform and frequency diversity parameters impose conditions on the transmit signals in terms of their time-domain and frequency-domain properties.

A. Spatial Diversity Constraint

Achieving spatial diversity intuitively means to make different transmitters get an uncorrelated view of the target independent of the transmit signals [8]. From equation, we see that the received signal component due to the *m*th transmitter is

$$r_m = C a(\theta) \alpha_m s_m \quad (6)$$

where C is the normalization constant to make the random variables $\alpha_m^2|^2$ a chi-square distribution. The uncorrelated target aspects result in uncorrelated fading coefficients and hence we have $E\{\alpha_m^* \alpha_{m+1}\} = 0$ which leads to $g_m^H g_{m+1}$ = 0. Hence uncorrelated aspects of target are seen by the transmitters if the columns of G are orthogonal. For small angles of ϕ_m and ϕ_{m+1} we have

$$\sin \phi_m - \sin \phi_{m+1} = d_t/R \quad (7)$$

which results in the following equality

$$g_m^H g_{m+1} = \sum_{p=0}^{P-1} e^{j2\Pi (d_t/R) p \Delta/\lambda} = 0 \quad (8)$$

For far enough target locations we have [8],

$$\frac{d_t \Delta}{R\lambda} \geq \frac{1}{P} \quad (9)$$

so that the orthogonality condition Eq. (8) is approximately met. This ensures that independent of the transmit signals type each transmitter has an uncorrelated view of the target.

B. Waveform and Frequency Diversity Constraint

Waveform diversity is implemented such that half of the total transmitters send orthogonal waveforms and other half transmit coherent ones. Hence the total transmit signal set comprises of ot_i, the orthogonal waveform from *i*th transmitter and co_j, the coherent waveform from *j*th transmitter where $i \in \mathbf{T}_1$ and $j \in \mathbf{T}_2$ such that $\mathbf{T}_1 \cap \mathbf{T}_2 =$ and $\mathbf{T}_1 \cup \mathbf{T}_2 = \mathbf{T}$, $\mathbf{T}$ = [0,1,...,$M-1$]. This ensures that some transmitters send orthogonal signals and the rest of them send coherent signals. Hence summing up, the following constraints are imposed on these waveforms:

(A) $\forall i, j \in \mathbf{T}_1; ot_i, ot_j$ are orthogonal
(B) $\forall i, j \in \mathbf{T}_2; co_i, co_j$ are not orthogonal
(C) $\sum_{i \in \mathbf{T}_1} ot_i, \sum_{i \in \mathbf{T}_2} co_i$ are orthogonal

The center frequency of the orthogonal set and coherent set are chosen to be different hence ensuring frequency diversity among the two sets of waveforms. This constraint is further analyzed in the following section in the regard of the matched filter design of signal detection in receivers.

C. Ambiguity Function Pair

Consider Eq. (6) which gives the received signal. Instead of designing a single ambiguity function for this final received signal as depicted in the end of above function we try to separate the orthogonal and coherent transmit signal trails in the received signal and create two different ambiguity functions for each. Because of the frequency diversity constraint which differentiates the center frequencies for the spectrum of both sets, we employ some of the receivers to filter out the orthogonal set and others to filter out the coherent set, there by separating the component of received signal from orthogonal transmit set from its counter part. Notice that frequency diversity constraint alone enabled this operation. Say $\mathbf{R}_1$ and $\mathbf{R}_2$ are the two sets of receivers that filter out orthogonal and coherent trails respectively. Hence by placing the receivers at small distances and uniformly we get the final received signal as $r = \sum_{n=0}^{N-1} r_n$.

The respective components filtered out by $\mathbf{R}_1$ and $\mathbf{R}_2$ can be given by

$$r_{ot} = \sum_{i \in \mathbf{T}_1} ot_i' \tag{10}$$

$$r_{co} = \sum_{j \in \mathbf{T}_2} co_j' \tag{11}$$

For each of these two received components matched filters are employed for detection of transmit signals. Hence on a similar lines we create two ambiguity functions each corresponding to the two received components as follows

$$\Psi_{ot}(\tau, f_r) = \int_{-\infty}^{\infty} r_{ot}(t) r_{ot}^*(t-\tau) e^{j2\Pi f_r t} dt \tag{12}$$

$$\Psi_{co}(\tau, f_r) = \int_{-\infty}^{\infty} r_{co}(t) r_{co}^*(t-\tau) e^{j2\Pi f_r t} dt \tag{13}$$

These ambiguity functions correspond each to the different waveform types we have as transmit signals and hence explore the aspect of waveform diversity. The orthogonal signals have better tracking ability and coherent signals give higher resolution in range detection, hence this new tool, calling it the ambiguity function pair, gives insight into both aspects simultaneously. Hence by looking at plots of this function we can simultaneously keep track of the target and calculate its range with higher resolution, in the sense this framework is adaptive to target tracking and range resolution. The following section evaluates this argument.

IV. Results and Discussions

A MIMO radar system with 10 transmitters and 10 receivers each having a uniform linear array configuration is considered. The target is assumed to have 20 scatterers uniformly placed at $\Delta = 5m$. Waveforms formed from Gaussian pulse trains satisfying the conditions (A, B, C) are used for both orthogonal and coherent sets. In the transmitter block 5 transmitters send the orthogonal set and rest 5 send the coherent set. The center frequency of the monopulse of orthogonal and coherent sets are respectively at 8GHz and 12GHz respectively and they satisfy narrowband properties. This above set of transmit signals hence diversity aspects in both waveform and frequency domains. Further the receivers are placed uniformly at d_r = 10m and 1000 samples are to be considered for processing at receiver.

Figure 1. shows the ambiguity function pair for this configuration plotted with respect to range of the target considering the average distance of receiver from transmitter to be 500km. These plots show that the ambiguity function for orthogonal set has better resolution than for coherent set at the target position (at $\lambda = 0$). Figure 2. compares the resolution of range along both x and y axis coordinates where xy plane contains the target and receiver block. We see that the coherent set ambiguity function performs better in detection of range. Now we displace the target by 100m without changing the transmitter block architecture, and redo the calculations resulting in a new ambiguity function pair. Figure 3. and 4. are the new pair plots with respect to the range resolution indicating that the orthogonal set ambiguity function has detected this target displacement (peaks in the magnitude at symmetric positions with respect to $\lambda = 0$). No inference on this can be detected from the coherent set ambiguity function plot. After calculating the displacement from this new plot of orthogonal set we use Eq. (9) to the new system framework abide by spatial diversity constraint and retransmit the waveforms to get higher range resolution his time from the coherent set ambiguity function plot. It means to say that this methodology adaptively tracks the target and give higher range resolution. This proves that imposing diversity constraints on the system and transmit signal architectures we can adaptively track the target position.

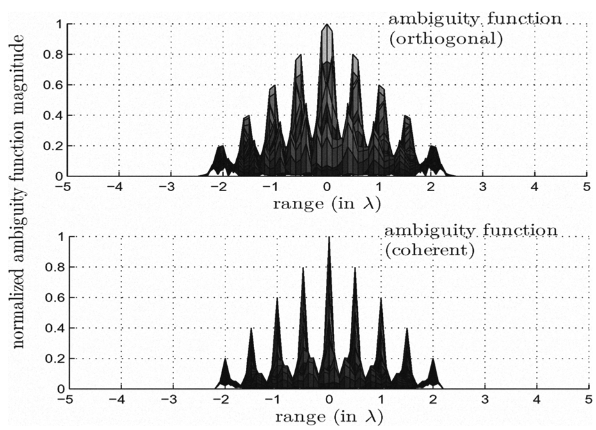

Fig. 1. Magnitude ambiguity function plot of orthogonal and coherent sets vs. range showing a peak at the target position. We see that coherent set detects target with higher resolution

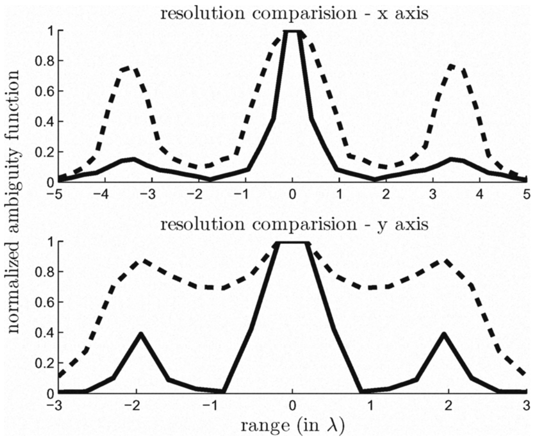

Fig. 2. Range resolution comparison plot for orthogonal and coherent sets along x and y axes (target and receiver block are assumed to be in xy plane) which shows coherent case performs better

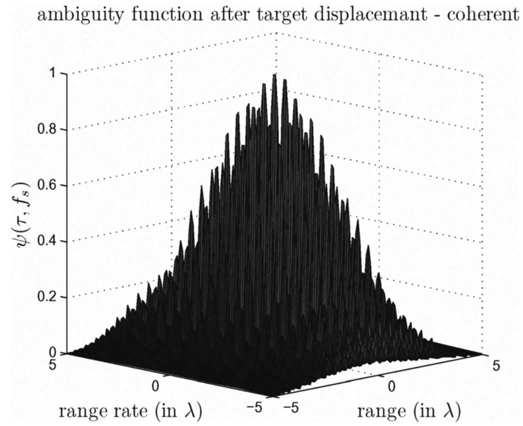

Fig. 4. Magnitude ambiguity function plot for coherent set after target is displaced by 2.5λ from its previous position where nothing can be inferred about this movement of target.

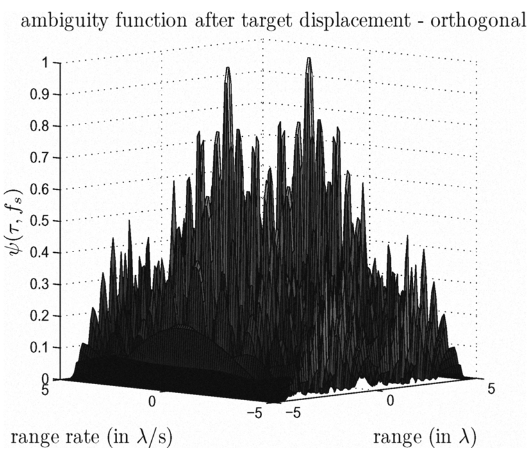

Fig. 3. Magnitude ambiguity function plot for orthogonal set after target is displaced by 2.5λ from its previous position showing this moved position and hence its tracking ability.

V. Conclusion

MIMO radar offers several diversity parameters that can be used to improve target detection and tracking when compared to conventional radar systems with single transmitter and receiver like bistatic system or single transmitter and receiver (SIMO) systems. This paper parameterizes these diversities mainly spatial, waveform and frequency diversities and evaluates a system employed with the resultant constraints. A new tool called ambiguity function pair is introduced to provide insight into the resultant performance criterion showing that using this tool MIMO radar systems employing diversity constraints can adaptively detect and track the target position. Future work may be to make a theoretical analysis of waveform and frequency diversities and developing system and tools with complete detection and tracking capability of complex targets with multi dimensional geometries.

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